Ballistic Pendulum Research Notes and Data Analysis Summary
June 5th – 11th, 2025 Zachary Hannan and Michael Sullivan, zhannan@solano.edu, zakslabphysics.com/ballistic

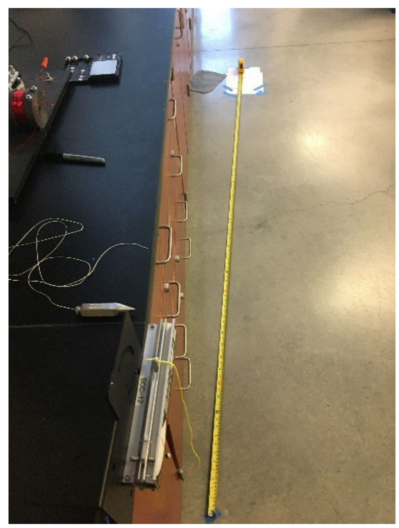
I. Collection of projectile range data and calculation of reference muzzle speed:

We set up a projectile launcher on the edge of a counter in the lab and clamped it in place with a C-clamp. We set the projectile launcher to an angle of zero as measured by the included protractor/plumb bob. We note that the protractor is quite small, and the plumb bob is crude (rough string with a piece of split shot clamped to it), and we decided to give the angle an uncertainty of 1° (smallest division on the instrument) as a result. This means the range formula needs to be reworked in terms of a general launch angle, so we can appropriately propagate error for the launch velocity.

A spot was marked on the floor using a proper plumb bob, directly below the indicated spot on the mini-launcher for which the ball loses contact with the plunger and begins its flight. We had to estimate a spot directly beneath the ball itself, and this required eye-balling a spot near the one marked by the plumb bob. As a result of this estimate, we adopt an uncertainty of 2mm on all range calculations. Note that because this uncertainty causes a systematic effect on every range measurement, we add it in quadrature with the statistical uncertainty in range. In any case, the uncertainty in range is ultimately swamped by the uncertainty in angle, as we see below.

The height was measured using a plastic meter stick graded in millimeters, and for this measurement we adopt an uncertainty of 1mm. Height was measured from the bottom of the ball from the release position to the floor.

mini-launcher setup showing protractor, spot marked on the floor beneath release point for the ball, plumb bob, meter stick and tape measure.
not shown: landing spot with carbon paper.



tape measure stretched to landing spot – white paper that we set carbon paper on during launches

Typical cluster of points (left) for a run of 10 shots, old run (right) scratched out.

We set the mini-launcher to its fastest setting (3 'clicks'). We collected data in three sets of 10 shots to minimize the chance of the ball landing in the same place twice. In all cases, we were able to distinguish 10 marks from the shots, but marks were overlapped significantly in some cases.

We also note that each trial contained a single outlier that was noticeably shorter than the other nine shots. We still included these points, as they were not outrageous, but it seems something goes wrong occasionally to slightly diminish the energy of the projectile (rattling in the barrel?). Again, it turns out the uncertainty in range is swamped by the uncertainty in launch angle, as we see below.

We processed the raw data for the range values in our range method spreadsheet[1]. A summary of collected data is presented below:

Launch Angle (θ): $0 \pm 1^{\circ}$

Height (h): 95.8 ± 0.1 cm

Mean Range (R): 254.6 cm

Standard Deviation (σ_R): 0.9345 cm

Uncertainty in Range: including the systematic 2mm uncertainty, we adjust the uncertainty to $\Delta R = .9557\text{cm}$ by adding the uncertainties in quadrature.

Note that we used a value of g with several significant digits, $9.80665\ \frac{\text{m}}{\text{s}^2}$ and we consider this an exact value in our calculations.

Using our range formula for the level launch, we obtain: $v_0 = R\sqrt{\frac{g}{2h}} = 2.546\text{m}\sqrt{\frac{9.80665\text{m/s}^2}{2 \cdot .958\text{m}}} \approx 5.760\text{m/s}$.

To propagate errors on the range method muzzle speed, we use the same spreadsheet[1]. To allow an analysis of error propagation including launch angle, we generalize to a symbolic expression for the muzzle speed as a function of launch angle and we obtain:

$$v_0 = \sqrt{\frac{R^2 g}{2\cos^2\theta\left(R\tan\theta + h\right)}}$$

We input this formula into Excel, then we individually vary R, θ and h by their indicated uncertainties, determining the corresponding changes in v_0 as Δv_{0R}, $\Delta v_{0\theta}$ and Δv_{0h}. These variations are then added in quadrature to obtain a value of uncertainty for v_0:

$$\Delta v_0 = \sqrt{\left(\Delta v_{0R}\right)^2 + \left(\Delta v_{0\theta}\right)^2 + \left(\Delta v_{0h}\right)^2}$$

see Hughes and Hase, *Measurements and Their Uncertaintites*, pg. 40-43 for an explanation of this numerical method[2]. A similar numerical method applies to all error propagation in these notes, and spreadsheets are provided in each case.

Note in our Excel formula we have used RADIANS to convert the degree measurement to radians for evaluation.

We obtain individual deltas:

$\Delta v_{0R} = .02162,\ \Delta v_{0h} = -.00300,\ \Delta v_{0\theta} = -0.12827$

We see that the uncertainty in muzzle speed is dominated by the uncertainty of 1 degree in angle! The other uncertainties are practically negligible, but we add them in quadrature anyway to get our uncertainty in muzzle speed:

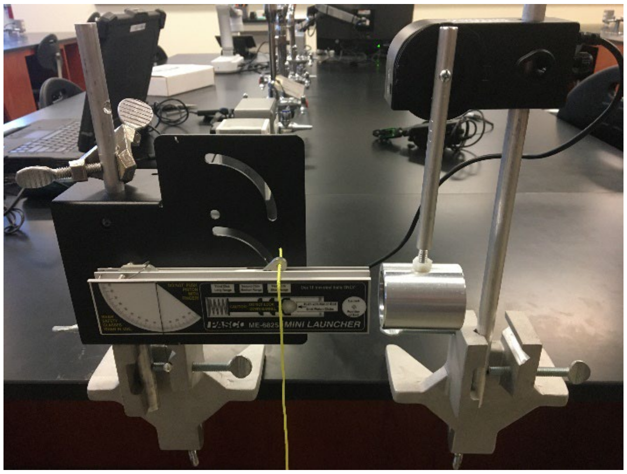
$\Delta v_0 = 0.13\text{m/s}$

We choose to keep two digits on our uncertainty (meaning the theta term actually completely defined the uncertainty here), and we state our range method muzzle speed value rounded to the same place value addressed by the uncertainty:

$$\boxed{v_0 \pm \Delta v_0 = 5.76 \pm 0.13 \text{ m/s}}$$

II. A. Collection of maximum angle data, aluminum ballistic pendulum.

We set up the ballistic aluminum ballistic pendulum with TPU insert together with the same mini-launcher at a new work station.



Aluminum pendulum ready for a ballistic pendulum run.

Top view: the angle we see while trying to align the gun and pendulum.

We weighed several ball bearings on our new .01g pocket scale and got mass values that bounced around between 16.33g and 16.35g. However, we later noticed this scale giving readings that fluctuated by about .05g during repeated measurements, and we decided to adopt an uncertainty of .05g for every mass measurement taken from this scale.

$m = 16.34 \pm .05\text{g}$ (mass of ball bearing projectile)

We proceeded to measure the maximum deflection angle of the pendulum, setting the mini-launcher again to its fastest setting (3 ‘clicks’). To get higher angular resolution, we set the sensor on “X4 mode”, which claims to set the resolution to a quarter of a degree, but the stated numbers in Logger Pro were rounded to the tenths place, making the resolution effectively 0.3 degrees.

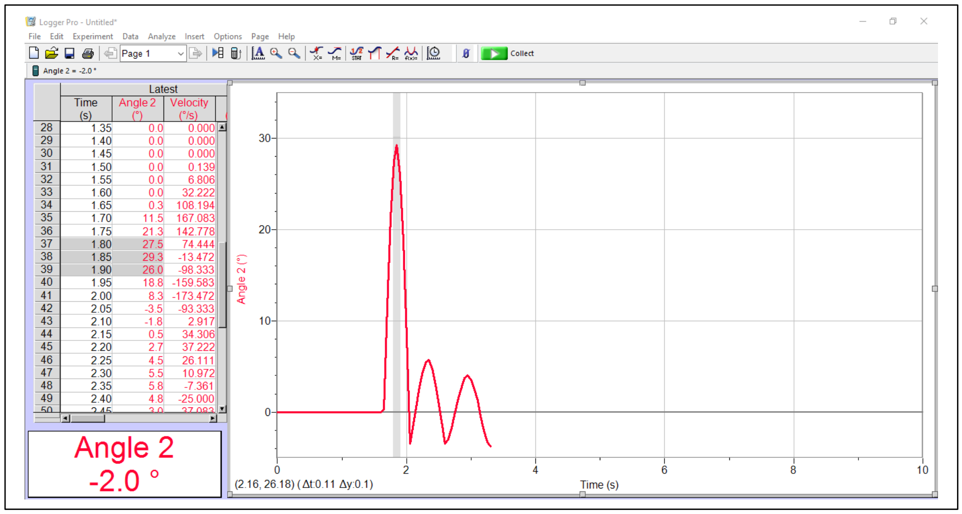
We initially set out to take 30 data points and use a standard deviation for uncertainty, however a practical matter emerged which led to a change in approach.

Here is the original data set:

Maximum Angle (degrees):

29.3	**29.3**
29.3	28.7
29.0	**29.3**
29.0	29.0
28.7	29.0
28.7	28.7
29.0	29.5
29.0	**29.3**
29.0	**29.3**
28.7	**29.3**
28.7	**29.3**
29.0	29.0
29.3	**29.3**
29.3	**29.3**
28.7	**29.3**

And a screenshot of a typical run in Logger Pro:



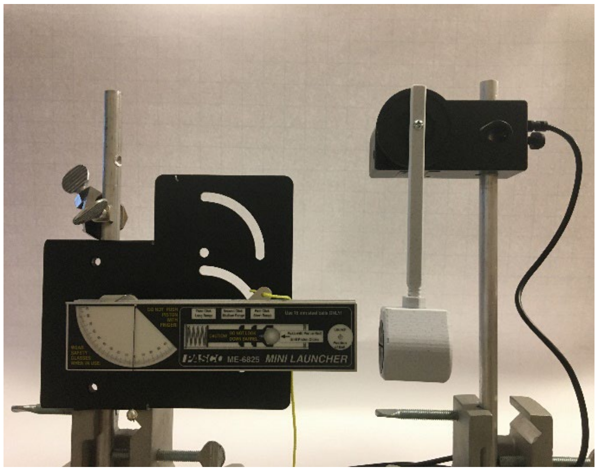
We noticed during the first several runs that our maximum deflection angle was decreasing systematically, and we realized that the gun and pendulum had drifted somewhat out of alignment, which should delete the component of muzzle velocity perpendicular to the plane of the pendulum's rotation, leading to a smaller maximum angle. Upon realigning the apparatus, we obtained another 29.3 degree reading. This continued throughout the process, and we observe that 7 of the last 8 impacts were 29.3 degrees as we improved our alignment methods. There is only one outlier above this value.

We conclude that the maximum angular deflection should not be treated as a sample from a normal distribution, but instead the most accurate value in terms of modeling the physics here should be the *maximum repeatable angular deflection* in the data set, indicating that the off-axis velocity component was near zero when the alignment was well-tuned. We are not confident in the high outlier, so that is discarded and we state our angular deflection as

$\theta_{\max} = 29.3 \pm 0.3^\circ$ (metal pendulum)

II. B. Collection of maximum angle data, 3D printed pendulum.

We repeated a similar procedure for the 3D-printed pendulum, and decided again to use the maximum repeatable angle as our most reliable measure of the pendulum's deflection.



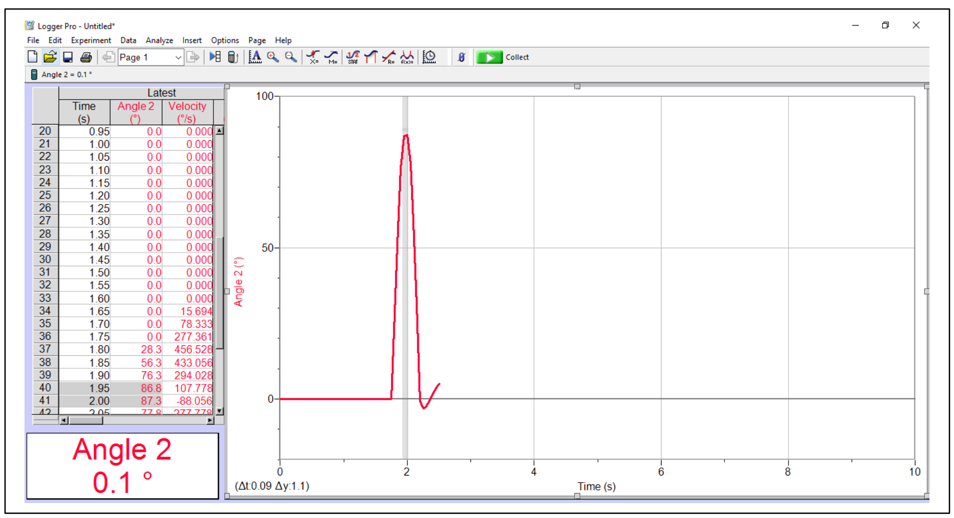
3d printed pendulum ready for a shot (sort of, alignment was off in the photo)

Here is the original data set:

Maximum Angle (degrees):

87.8	87.8
87.5	87.3
87.5	**87.5**
87.0	87.0
87.5	**87.5**
87.3	**87.5**
86.5	87.0
88.0	86.8
86.8	87.3
86.5	**87.5**
87.5	87.8
86.5	87.3
87.3	**87.5**
87.0	**87.5**
87.5	87.3

And a screenshot of a typical run in Logger Pro:



Using our strategy of keeping the *maximum repeatable angle of deflection*, we have to discard four measurements above 87.5 degrees, but we think 87.5 is the clear maximum repeatable angle, because it appears 11 times in the data set.

$\theta_{max} = 87.5 \pm 0.3^{\circ}$ (3D printed pendulum)

III. A. Center of Mass Data, Aluminum Pendulum

We take several measurements for the calculation of center of mass of the metal pendulum. Length measurements were taken with Vernier calipers and mass measurements with our 0.01g pocket scale. We did not use the full precision of the calipers because measurements were a little awkward, as indicated, and we did not use the full precision of the scale due to the roughly 0.05g fluctuations we observed with repeated measurements.

Note that we had to get the magnet as far away from the scale as possible to avoid an overestimate of mass caused by the magnet interacting with the scale internals. In the picture, the TPU insert is removed and stacked on the catcher with the magnet pointed up.

Balancing the metal rod to find its center of mass.

Measuring mass of the catcher, magnet and ball.

Measuring mass of the rod.

Distance from rotation axis to center of catcher:

We were able to insert the blade of the caliper into the catcher insert to reduce the tilt of the calipers on this measurement, but there was still a perceptible tilt, and the center of the mounting hole and center of the catcher fins had to be estimated. We gave this measurement a 2mm uncertainty due to its awkwardness:

$r_C = 14.3 \pm 0.2$ cm

Note that CAD models of the 3D printed pendulum put the center of mass of the catcher (ball and magnet included) less than 1mm above the center of the catcher. We expect the metal model to have a center of mass of the catcher slightly *below* the center, since a small amount of material was milled from the top surface to create a mating surface for the nut. In both cases, we're confident the offset of the center of mass is within our measurement uncertainty from the calipers.

Distance from rotation axis to center of mass of the rod:

We balanced the rod on another aluminum rod (used as a fulcrum) and marked the balance point, then measured from the mounting hole (rotation axis) to the balance mark. We estimate a 1mm uncertainty on the placement of the balance point mark and obtain:

$r_R = 5.1 \pm .1$ cm

Note: we included the nylon nut on the rod, since the nylon nut is in place during the experiment.

Mass of the catcher:

Simple measurement on the pocket scale (ball included):

$m_C = 147.13 \pm .05$ g

Mass of the rod:

Again, placed on the scale, including the nylon nut:

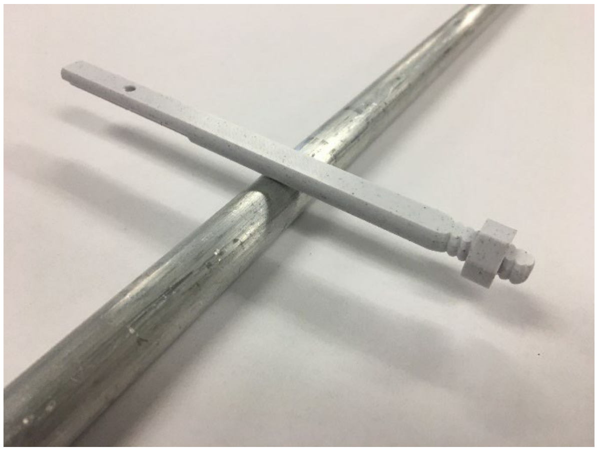
$m_R = 23.62 \pm .05$ g

We calculate r_{cm} and perform numerical error propagation in our center of mass spreadsheet[3] and obtain:

$$\boxed{r_{cm} = 13.03 \pm 0.17\text{cm (center of mass/metal pendulum)}}$$

III. B. Center of Mass Data, 3D Printed Pendulum

All procedure notes are the same for the measurements taken from the 3D printed pendulum. Results are as follows:



Balancing the 3D print rod to find its center of mass.

Finding mass of 3D catcher, ball and magnet.

Finding mass of 3D rod.

Distance from rotation axis to center of catcher:

$r_C = 14.1 \pm 0.2$ cm

Distance from rotation axis to center of mass of the rod:

$r_R = 5.6 \pm .1$ cm

Note: the nut should play a larger role in shifting the center of mass of the 3D printed rod.

Mass of the catcher:

$m_C = 53.81 \pm .05$ g

Mass of the rod:

$m_R = 7.14 \pm .05$ g

We calculate r_{cm} and perform numerical error propagation in our center of mass spreadsheet[3] and obtain:

$$\boxed{r_{cm} = 13.10 \pm 0.18\text{cm} \ \text{(center of mass/3D pendulum)}}$$

IV. A. Small Oscillations Method, Aluminum Pendulum

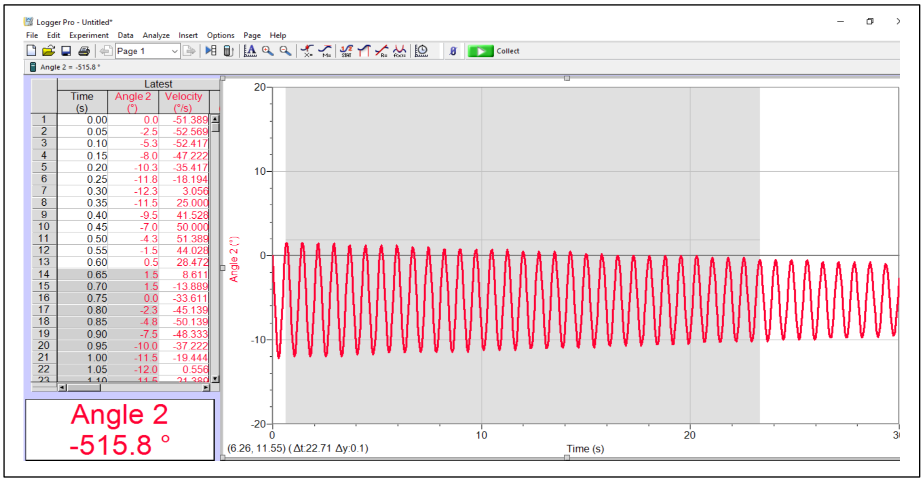
Measuring Period:

We set the pendulum (including the ball) into small oscillations with an amplitude of about 5 degrees. We recorded the data from the rotation sensor and found a total time of 22.71s for 30 periods or a period of .757s. Since this is *not* a set of 30 independent trials, we take a reasonable absolute uncertainty of 0.1s and divide it by 30 to get about .003 s. (Logger Pro sampling rate is .05s, and we don't trust our placement of the cursor at the peaks of the position function, so 0.1s is a safe uncertainty on the total time) .

We calculate the period:

$T = 0.757 \pm .003\text{s}$ (metal pendulum)

And here's what the data collection looks like in Logger Pro:



Calculating moment of inertia:

Recall from our paper, that moment of inertia is computed as $I = \dfrac{Mgr_{cm}T^2}{4\pi^2}$. Where big M is the combined mass of the rod and catcher (ball included).

M (mass of the rod, catcher and ball) should be measured all at once instead of adding the mass of the catcher and rod (we avoid additional propagation of error by handling it this way).

We measure the total mass of the pendulum/rod/ball combination to avoid propagating errors on individual masses, and we get:

$M = 170.73 \pm 0.05\text{g}$

Calculation of moment of inertia and error propagation is performed in our period method spreadsheet[4] and we obtain:

$$\boxed{I = 0.003167 \pm 0.000048 \text{ kgm}^2 \text{ (metal pendulum)}}$$

Note that we consider our value of g to be exactly $9.80665\dfrac{\text{m}}{\text{s}^2}$ in the spreadsheet, because we kept so many digits on the reference value.

Calculating muzzle speed:

Finally, we use the muzzle speed formula from our paper $v_0 = \frac{1}{mr_C}\sqrt{2MIgr_{cm}\left(1-\cos\theta_{\max}\right)}$:

We insert all measured values and propagate error in our muzzle speed spreadsheet[5] and we obtain our final muzzle speed:

$$\boxed{v_0 \pm \Delta v_0 = 5.69 \pm 0.11 \frac{\text{m}}{\text{s}} \text{ (metal/period method)}}$$

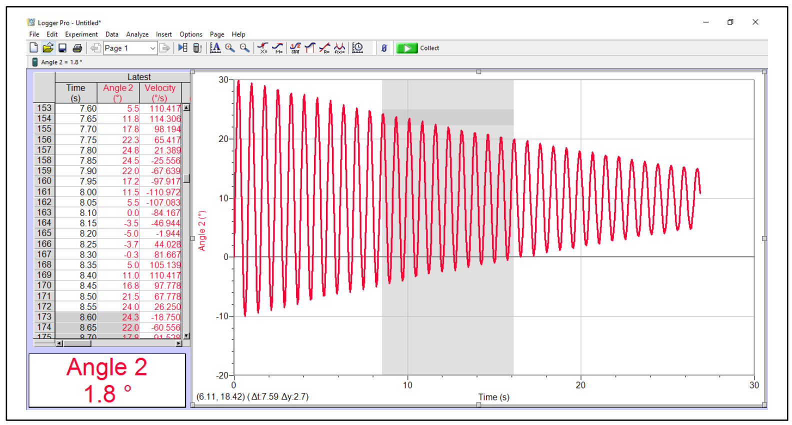
Note that this is a 1.2% difference compared to the reference value, computing the percent difference as $\frac{v_{01}-v_{02}}{\left(v_{01}+v_{02}\right)/2}$, and each velocity value is firmly within the confidence interval of the other.

IV. B. Small Oscillations Method, 3D print pendulum

Calculating period:

We note that friction played a more prominent role in the behavior of the lighter 3D printed pendulum when we were calculating the period of small oscillations. The amplitude decreased too significantly to keep 30 oscillations, and we noted that the period changed significantly from the $15^{\circ}-10^{\circ}$ amplitude regime to the $5^{\circ}-2^{\circ}$ regime, with the small amplitude data showing some distortion in the angle as a function of position curve presumably caused by a rough spot in the bearing races that produced a slight "click" with each oscillation. We decided to find our period by averaging over 10 oscillations in the $15^{\circ}-10^{\circ}$ regime. Obtaining a total time of 7.59 seconds for 10 periods, or 0.759 seconds for the period. Again, we use an uncertainty of 0.1s for the total time, yielding an uncertainty of .01s for the period:

$T = 0.759 \pm .01\text{s}$ (3D printed pendulum)



Calculating moment of inertia:

We measure the total mass of the pendulum/rod/ball combination to avoid propagating errors on individual masses, and we get:

$M = 60.94 \pm 0.05\text{g}$

Calculation of moment of inertia and error propagation is performed in our period method spreadsheet[4] and we obtain:

$$I = 0.001142 \pm 0.000034\ \text{kgm}^2\ \text{(3D print pendulum)}$$

Note that we consider our value of g to be exactly $9.80665\frac{\text{m}}{\text{s}^2}$ in the spreadsheet, because we kept so many digits on the reference value.

Calculating muzzle speed:

We calculate muzzle speed and propagated error as before, in our muzzle speed spreadsheet[5] and we obtain:

$$v_0 \pm \Delta v_0 = 5.68 \pm 0.12\frac{\text{m}}{\text{s}}\ \text{(3D/period method)}$$

We obtain a 1.4% difference, and the uncertainty is slightly higher than the metal pendulum results.

V. A. Atwood machine method (rotation method), Aluminum Pendulum

We set up a “rotational Atwood machine” using the metal pendulum. We ultimately had to use a threaded spacer to get clearance between the rotating pendulum and the attached pulley that guides the string off the rotary motion sensor hub (this wasn’t strictly mandatory, but it improved the angle between the sensor hub and the pulley which reduced friction and reduced the probability of the string slipping off the pulley). Note that the pulley attachment comes with the Vernier rotational motion accessory kit (Vernier part AK-RMV), and not all labs will have this part available.

We went through a very long process of troubleshooting this part of the procedure, initially obtaining moment of inertia values that were significantly higher than expected. Instructors should be aware that this rotation method is problematic, but we include our troubleshooting process to provide ideas for how you might reduce error in a teaching setting:

1. One source of uncertainty is what happens when the string unwinds to the end. We tried the following:

(a). taping the string securely enough to not fall off the hub at the end of the process but instead re-wind onto the spool after bottoming out – this method was discarded because the rapid change from increasing to decreasing angular velocity made it difficult to spot maximum angular velocity in the data; i.e., the sampling rate of the sensor introduced too much error. We also observed runs where the tape *partially* came off, effectively lengthening the string.

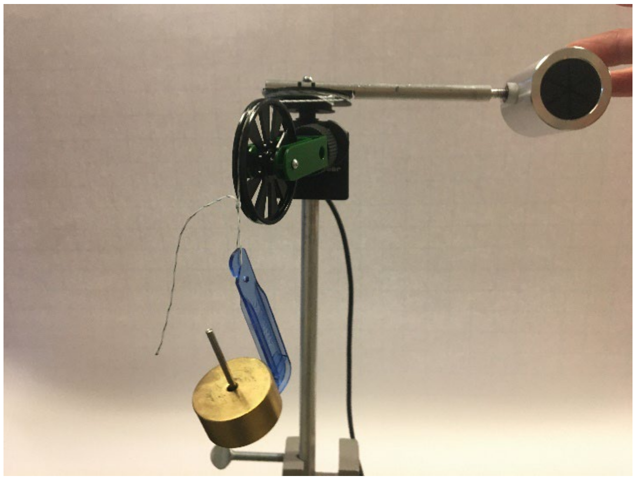
(b). taping lightly enough for the string to pop off at the end of the process – this method was discarded because it wasn’t clear if the tape popped off a little early, effectively decreasing the value of y for the process.

(c). allowing the mass to come to rest on the ground and using the max angular velocity measurement at that moment -- This resulted in a “smooth maximum” angular velocity because there was no rapid change in omega and less ambiguity with regard to the effective length of string. We marked a spot on the string where it lost contact with the hub before running the experiment, slowly unwound to the mass contact point with the ground and marked a similar spot on the string, then measured the length between spots after collecting angular velocity data. The string was about 15 cm longer than necessary, ensuring the tape would not fail during the experiment because we had a couple extra turns on the rotary motion sensor hub when the mass reached its lowest point.

We ultimately adopted a 1cm uncertainty on the length of string due to the awkwardness of measuring the spot that loses contact with the hub for the start and finish positions, and this added uncertainty of a similar size to other sources in the error propagation calculation (same magnitude as the uncertainty in angular velocity, but about twice as large).

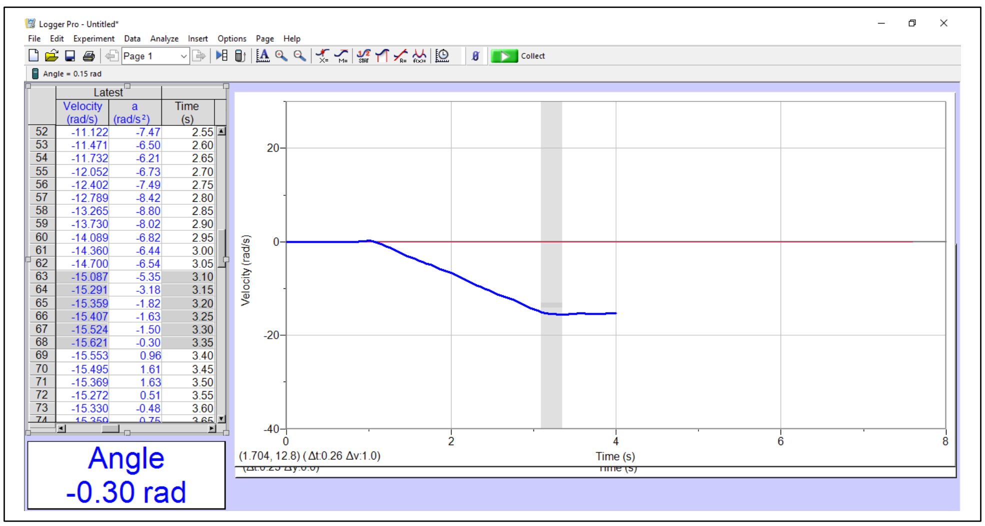
2. The second major source of error was actually friction in the rotary motion sensor bearings. We discovered this by experimenting with different lengths of string (y), and it turned out that the final angular velocity was not just systematically lower than expected (leading to a higher calculated MOI and therefore a higher muzzle speed) – in fact, the problem got worse as we used longer and longer pieces of string (up to about 1.7m). Our best guess is that the accumulated energy losses due to friction in the bearings are to blame here, but we also observed a lot of wobbling in the rod framing, which we can also blame for some energy losses (we tested for this by slowing the process down, using a smaller hanging mass to reduce wobbling – but the length of string seemed to dominate the error we were seeing).

After nailing down the length of the run as the main cause of obtaining moment of inertia values that were far too high, we settled on using a very short run of about 40cm for this part of the lab. We accomplished this by stacking boxes on the floor, so the hanging mass came to rest on a box at the end of the run, giving us smooth angular velocity data with a clear maximum value.



aluminum pendulum set for a rotation method run

We include our Logger Pro data for the metal pendulum below (note that the negative angular velocity is just an arbitrary sign choice that we didn't bother to reconfigure):



We highlighted a short region near the point where the sensor deviated from linearity, indicating the hanging mass had touched down, then we grabbed the maximum angular speed of about 15.6 rad/s from that region.

Unfortunately, we didn't gather sufficient data to estimate the uncertainty in the measurement of angular velocity in a robust way. We did, however, obtain a small data set of repeated trials using the "tape popping off method" earlier in the day. After refining that technique (using a longer string) we obtained: 37.3, 37.2, 37.08, 37.3 (rad/s), giving us a sense of how consistent the rotation sensors are. The standard deviation of this data set is about 0.1rad/s, so we adopt an uncertainty of 0.1rad/s on our angular velocity data, which justifies rounding the angular velocity measurement to the tenths place. This should be considered a conservative value, because the "mass coming to rest on a box" method should be more reliable. This uncertainty contributed to the uncertainty in moment of inertia at the same order of magnitude as the uncertainties in y and d.

Recall from our paper:

"We wind a length of string y around the rotary motion sensor's hub (diameter d), hang a mass m_h off the string using the included pulley and employ a conservation of energy analysis. We obtain an expression for pendulum moment of inertia in terms of final angular speed ω_f, which we can determine from the data output of the sensor:

$$I = m_h\left(\frac{2gy}{\omega_f^2} - \frac{d^2}{4}\right) \quad (1)"$$

Our measurements are as follows:

$m_h = 104.53 \pm 0.05\text{g}$ (measured with .01g pocket scale)
$y = 40 \pm 1\text{cm}$ (measured spot to spot on the string after the run)
$\omega_f = 15.6 \pm .1$ rad/s (based on variation in an earlier repeated trial)
$d = 4.82 \pm .01\text{cm}$ (caliper measurement of hub diameter, averaged over values with string wrapped on it and without, did not use full precision of the calipers due to ambiguity of caliper blades pushing the string aside)

In our rotation method spreadsheet[6] we calculate the resulting moment of inertia and uncertainty, obtaining:

$$I = .003309 \pm .000095 \text{ kgm}^2 \text{ (metal pendulum/rotation method)}$$

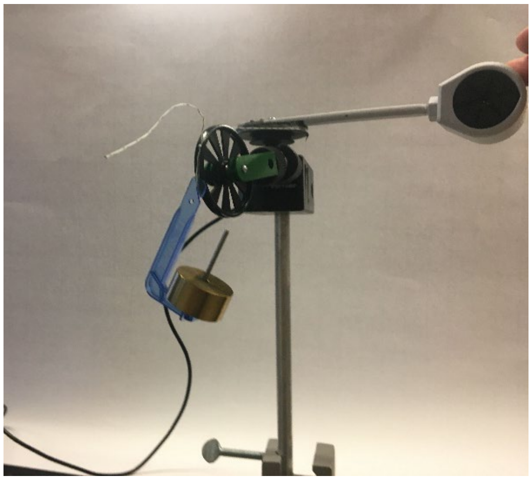
In our muzzle speed spreadsheet[5], we calculate the resulting muzzle speed, obtaining:

$$v_0 \pm \Delta v_0 = 5.82 \pm 0.14 \frac{\text{m}}{\text{s}} \text{ (metal pendulum/rotation method)}$$

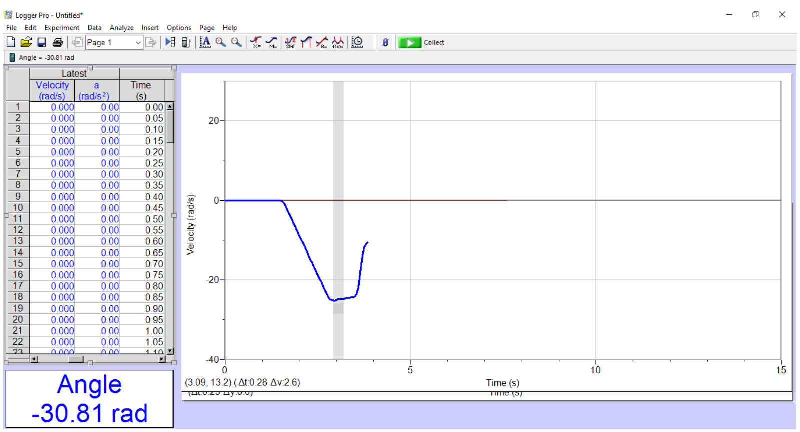
This is approximately a 1.0% difference, and notably, it's higher than the value obtained using the period method or the projectile shot. Again, we believe this is due to energy losses caused by friction in the bearings, which leads to a higher value of moment of inertia, which then calculates back to a larger muzzle speed.

V. B. Atwood machine method (rotation method), 3D Printed Pendulum

We used all the same procedures developed for the metal pendulum and obtained the following angular velocity data for the 3D print pendulum:



3D print pendulum set for rotation method run



The final angular velocity isn't highlighted in the screenshot, but it was measured as 25.3 rad/s with the same uncertainty of 0.1 rad/s adopted based on our earlier repeated set of 4 trials with a longer string. Other data is listed below:

$m_h = 104.53 \pm 0.05\text{g}$

$y = 40 \pm 1\text{cm}$

$\omega_f = 25.3 \pm .1 \text{ rad/s}$

$d = 4.82 \pm .01\text{cm}$

In our rotation method spreadsheet[6] we calculate the resulting moment of inertia and uncertainty, obtaining:

$$I = .001220 \pm .000034 \text{ kgm}^2 \text{ (metal pendulum/rotation method)}$$

In our muzzle speed spreadsheet[5] we calculate the resulting muzzle speed, obtaining:

$$v_0 \pm \Delta v_0 = 5.87 \pm 0.12 \frac{\text{m}}{\text{s}} \text{ (3D print pendulum/rotation method)}$$

This is approximately a 1.9% difference, and notably, it's higher than the value obtained using the period method or the projectile shot.

Note again that we know this answer is too high due presumably to the compounding energy losses due to friction that we observed by experimenting with larger values of y, but we didn't want to take a shorter run than 40 cm because that would amplify the relative uncertainty in string length. Instructors are advised to experiment with shorter runs to see if better agreement can be obtained, but our conclusion is that this rotation method approach has a systematic lean toward overestimating moment of inertia. This problem is worse for the lighter 3D print pendulum. We conclude that the period method is superior because the period of oscillation remains the same as the amplitude of oscillations decreases (within the 15-5 degree regime). However, the rotation method provides students an opportunity to apply energy conservation to a system with both rotating and translating parts, so instructors may want to use this method for its pedagogical value.

VI. A. Piecewise Method (Metal Pendulum)

For the piecewise method estimate of moment of inertia, we modeled the pendulum as a uniform rod from the rotation axis to the mating surface with the catcher, in addition to a point mass representing the catcher at its geometric center. Note that we neglect the part of the rod on the opposite side of the mounting screw at the rotation axis, using the argument that the aluminum rod was milled down to very nearly half its original thickness over the mating surface with the rotary motion sensor hub. So, we imagine putting the flattened section on the opposite side of the mounting screw together with the flattened section on the catcher side of the mounting screw to obtain a uniform rod rotating exactly about its end. In addition, we assume the nut on the threaded end of the rod compensates for the material removed by cutting the threads.

To get a rough sense of how much uncertainty is introduced by modeling the catcher as a point mass, we decided to compare to the numbers obtained by modeling it as a thick cylindrical shell (with no ball or magnet) together with the ball and magnet represented as a point mass at the center of the catcher.

The analytic calculation of the MOI for a thick cylindrical shell rotating about an axis perpendicular to its symmetry axis + application of the parallel axis theorem is significant, and might make a good guided problem for a calculus based physics course, depending on the instructor's level of emphasis on MOI integrals. Here, we just looked up a formula for the result and found that the moment of inertia can be modeled as follows:

$$\frac{1}{4}m_{C\,no\,ball}(a^2+b^2)+\frac{1}{12}m_{C\,no\,ball}h^2+\underbrace{m_{C\,no\,ball}r_C^2+m_{ball/mag}r_C^2},$$

where the third term is due to the parallel axis theorem, and the bracketed portion is simply the point mass approximation of the catcher with ball and magnet residing at its center.

The first two terms capture the underestimate of the point-mass approximation, where a and b are the inner and outer radii and h is the length of the cylinder; i.e., in the first two terms, we're just adding in the rotation of the shell about its own perpendicular axis, which should be a relatively small contribution.

The first two terms give us the approximate underestimate we commit by using a point mass approximation for the whole catcher, ball and magnet included, so we calculate

$$\frac{1}{4}m_{C\,no\,ball}(a^2+b^2)+\frac{1}{12}m_{C\,no\,ball}h^2$$

where:

$m_{C\,no\,ball} \approx 124.83\text{g}$ - catcher/insert mass without magnet and ball
$a \approx .016\text{m}$
$b \approx .022\text{m}$
$h \approx .050\text{m}$
$r_C \approx .143\text{m}$

We find that the correction term is about .00005kgm^2. We use this value as a stand-in for the uncertainty introduced by all the inconsistencies in geometry for the pendulum and the crude representation as a uniform rod and point mass – the fact that the metal is thicker at the back of the catcher, and a washer is epoxied into the back, the fact that a section was milled off the top to provide a seat for the nylon nut, and so on.

We add this in quadrature with the uncertainty due to all other measurements to obtain an uncertainty of .00098kgm^2

A further complication to the piecewise method is that it is the only method that doesn't automatically include the moment of inertia of the rotary motion sensor hub. However, we justify ignoring the moment of inertia contribution of the hub as follows:

This hub is essentially a stack of plastic disks (pulleys) of different diameters, and we estimated the moment of inertia through a series of overestimates and underestimates to simplify the geometry: the middle disk could be used to fill the cavity in the larger disk, so we can consider the hub as a single larger disk. However, this slightly underestimates the moment of inertia because the middle disk is slightly larger in diameter than the cavity. Thus, we slightly overestimate the radius of the larger disk by using the diameter as measured from the edges of the channel that guides the string, rather than the surface the string rides on. We overestimate the moment of inertia by ignoring the roughly 1.4cm central hub that runs through the disks, essentially spreading its mass over the single disk in our calculation, but again we underestimate by ignoring the plastic tabs at the outer edge of the large disk and assuming *that* mass is uniformly distributed as well. In the end, we model the hub as a single disk with outer diameter 52.1mm and mass 7.870g, giving a rough contribution of .000003 kgm^2 which is negligible in comparison to our existing error bars on the total moment of inertia.

Here's the relevant data for the simple piecewise approximation of MOI as a uniform rod together with a point mass at the location of the center of the catcher:

$r_C = 14.3 \pm 0.2$ cm (previous measurement from rotation axis to center of catcher)
$L_R = 12.1 \pm .1$ cm (measured to the mating surface with catcher)
$m_C = 147.13 \pm .05$ g
$m_R = 23.62 \pm .05$ g

We use common moment of inertia formulas to get the total MOI as:

$$\frac{1}{3} m_R L_R^{\,2} + m_C r_C^2$$

Calculations and error propagation are performed in our piecewise method spreadsheet[7] to obtain the moment of inertia:

$$\boxed{I = 0.003124 \pm .000098 \text{ kgm}^2 \text{ (metal pendulum/piecewise method)}}$$

Once again, we calculate muzzle speed using our muzzle speed spreadsheet[5] and obtain:

$$\boxed{v_0 \pm \Delta v_0 = 5.65 \pm 0.14 \frac{\text{m}}{\text{s}} \text{ (metal pendulum/piecewise method)}}$$

This is only a 1.9% difference compared to the range method, despite the crude estimate of the catcher as a point mass, and the confidence interval still captures the reference value of 5.76m/s.

VI. B. Piecewise Method (3D printed Pendulum)

Note that the 3D printed pendulum does not have the same reduction in width of the rod across the hub mating surface, so our procedure of taking the half-width sections of the rod and pasting them together to get a uniform rod isn't as well justified. We expect this to have a very small effect, since we're taking the part of a thin rod closest to the rotation axis. Note that our approximation of error using the MOI for a thick shell is even more flawed because the mass of the catcher insert plays a more prominent role due to the light weight of the plastic catcher.

We repeat the approximation of the underestimate committed by the point mass approximation as an estimate of the new uncertainty introduced in this method:

$$\frac{1}{4}m_{C\,no\,ball}(a^2+b^2)+\frac{1}{12}m_{C\,no\,ball}h^2$$

where:

$m_{Cno\,ball} \approx 31.55\text{g}$ - catcher with ball and magnet removed
$a \approx .0159\text{m}$
$b \approx .0218\text{m}$ - note this is an average over different parts of the catcher because it's not symmetric (there is more material at the top)
$h \approx .041\text{m}$
$r_C \approx .141\text{m}$

This time, our systematic underestimate is about .00001kgm^2, and we again use this as an estimate of the additional uncertainties introduced in our crude representation of the pendulum as a uniform rod together with a point mass. After adding this in quadrature with the existing uncertainties in measurements in our piecewise method spreadsheet[7] this has almost no impact on our final estimate of uncertainty.

Relevant data:
$r_C = 14.1 \pm 0.2$ cm (previous measurement from rotation axis to center of catcher)
$L_R = 11.77 \pm .1$ cm
$m_C = 53.81 \pm .05$ g
$m_R = 7.14 \pm .05$ g

And again we run the numbers through our piecewise approximation:

$$\frac{1}{3}m_R L_R{}^2 + m_C r_C^2$$

Calculations and error propagation are performed in our piecewise method spreadsheet to obtain the moment of inertia:

$$\boxed{I = 0.001103 \pm .000031\ \text{kgm}^2 \text{ (3D printed/piecewise method)}}$$

Once again, we calculate muzzle speed using our muzzle speed spreadsheet[5] and obtain:

$$\boxed{v_0 \pm \Delta v_0 = 5.58 \pm 0.12\frac{\text{m}}{\text{s}} \text{ (3D pendulum/piecewise method)}}$$

This yields a 3.2% difference compared to the range method. Again, as a pedagogical tool, this part of the lab is open-ended and interesting to the students, so we think it's worth keeping to reinforce the idea of how moments of inertia add, if that's the point the instructor chooses to emphasize.

Note that this interval narrowly misses capturing the reference value of 5.76m/s, but the confidence intervals for the two measurements do overlap significantly.

We reiterate that the period method takes everything into consideration with respect to moment of inertia in one simple measurement that is robust against the effects of friction or any geometric estimates of moment of inertia, so that really is the superior method of measuring I if accuracy is the key concern. For the 3D printed pendulum, it's harder to make the case that the (incorrect) linear momentum method produces significantly less accurate results than the angular momentum method. To clearly highlight the failure of the linear momentum method, we recommend using the period method with the 3D printed pendulum.

VII. A. Naïve linear momentum method (Aluminum Pendulum)

We finish with a quick calculation of muzzle speed using the naïve approach of assuming linear momentum is conserved. Apparently this was commonly done with the so-called "Blackwood pendulum", which generated so much discussion in The Physics Teacher and American Journal of Physics in the 70s and 80s.[8–11] We would have preferred to just approximate the linear momentum conservation method by saying the mass of the rod is negligible, but we decided to stick with tradition of acting as if the total mass is concentrated in the catcher.

We obtain the formula $v_0 = \frac{M}{m_b}\sqrt{2gr_c\left(1-\cos\theta_{\max}\right)}$ from pretending all the mass is located at the point of impact and linear momentum is conserved.

Relevant data:

$M = 170.73 \pm .05\text{g}$ - mass of pendulum with ball
$m_b = 16.34 \pm .05\text{g}$
$r_c = 14.3 \pm 0.2\text{cm}$
$\theta_{\max} = 29.3 \pm 0.3^0$

Muzzle speed: $v_0 \pm \Delta v_0 = 6.26 \pm .08\text{m/s}$ (linear momentum/metal pendulum)

We processed all the data in our linear momentum method spreadsheet[12]. This result is about an 8.3% difference compared to the reference value of 5.76m/s, and there is clear separation between the two confidence intervals.

VII. B. Naïve linear momentum method (3D Printed Pendulum)

$M = 60.94 \pm .05\text{g}$
$m_b = 16.34 \pm .05\text{g}$
$r_c = 14.1 \pm 0.2\text{cm}$
$\theta_{\max} = 87.5 \pm 0.3^0$

Muzzle speed: $v_0 \pm \Delta v_0 = 6.07 \pm .05\text{m/s}$ (linear momentum/3D printed pendulum)

Again, error analysis was done in our linear momentum spreadsheet[12]. This result is about a 5.2% difference compared to the reference value of 5.76m/s, and there is clear separation between the two confidence intervals.

Note that before we began this investigation, a primary concern was whether or not the 3D printed pendulum would show a significant difference between linear momentum and angular momentum methods (because the 3D printed rod is a smaller percent of total mass of the apparatus). This result shows that the 3D printed version still reveals the problem with using a linear momentum approach, so instructors choosing to 3D print this apparatus can still include this contrast in their lab procedures, but be warned that using the piecewise method of computing *I* can muddy the waters on this distinction.

[1] "Spreadsheet '1. range method,'" (2025).

[2] Hughes, and Hase, *Measurements and Their Uncertainties* (Oxford University Press, 2010).

[3] "Spreadsheet '2. center of mass,'" (2025).

[4] "Spreadsheet '3. period method MOI,'" (2025).

[5] "Spreadsheet '4. muzzle speed from MOI,'" (2025).

[6] "Spreadsheet '5. rotation method MOI,'" (2025).

[7] "Spreadsheet '6. piecewise MOI,'" (2025).

[8] T.R. Sandin, "Nonconservation of Linear Momentum in Ballistic Pendulums," American Journal of Physics **41**(3), 426–427 (1973).

[9] P.D. Gupta, "Blackwood pendulum experiment and the conservation of linear momentum," American Journal of Physics **53**(3), 267–269 (1985).

[10] E. Wicher, "Ballistics pendulum," American Journal of Physics **45**(7), 681–682 (1977).

[11] A. Sachs, "Blackwood pendulum experiment revisited," American Journal of Physics **44**(2), 182–183 (1976).

[12] "Spreadsheet '7. linear momentum method,'" (2025).